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Symmetry in exotic nuclei

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Abstract. Symmetries have played an important role in the elucidation of the structure of nuclei and will continue to do so for exotic nuclei. As an example, an application of pseudo-$SU(4)$ symmetry is discussed. It can be used as a starting point for a boson model that includes $T = 0$ as well as $T = 1$ bosons (IBM-4); applications are presented for $N = Z$ nuclei from ^{58}Cu to ^{70}Br.

PACS. 21.60.Fw Models based on group theory

1 A brief history of symmetry in nuclei

Symmetry considerations have played an important role in the development of nuclear physics. Already in 1932, at the inception of the discipline, the observed similarities between the proton and the neutron were interpreted by Heisenberg [1] in terms of an "isotopic" symmetry, the origin of which subsequently was related by Wigner [2] to the charge independence of the strong force. Since then, the use and application of symmetries in nuclear physics have gone from strength to strength. The most important developments include Wigner's $SU(4)$ supermultiplet model [2] which extends Heisenberg's idea to isospin *and* spin, Racah's $SU(2)$ pairing model [3] leading to the concept of seniority, Elliott's $SU(3)$ model [4] which provides an understanding of rotational band structures in the context of the spherical shell model and the $U(6)$ interacting boson model of Arima and Iachello [5] which gives a unified description of collective structures observed in nuclei.

These different models, which were developed over a period of more than half a century, can be understood from a common perspective using the concept of dynamical symmetry or spectrum-generating algebra (for a recent review, see ref. [6]). This approach is formulated rigorously in terms of the theory of Lie algebras and can be characterised in words as follows. Given a system of interacting particles (bosons or fermions) a definite mathematical procedure exists to construct a set of commuting operators which supply the quantum numbers of a classification scheme. Furthermore, to each set of commuting operators there corresponds a class of many-body Hamiltonians which can be solved analytically simply by requiring that they be written in terms of these commuting operators.

A point that should be emphasised is that this procedure is generic and equally valid for bosons and for fermions, as indeed it is for mixed systems of bosons and fermions. Thus, for example, the applications of the interacting boson model to even-even nuclei [7] could be extended naturally to odd-mass nuclei by introducing an interacting boson-fermion model [8]. The fact that bosons and fermions can be treated alike has led to the formulation of a supersymmetric model [9] which provides a simultaneous description of "boson-like" (even-mass) and "fermion-like" (odd-mass) nuclei. Nuclear supersymmetry has been tested extensively and confirmed in a number of cases in the 1980s. Recent advances in detector resolution have made it possible to propose tests of dynamical supersymmetry in even more complicated odd-odd nuclei, and were successful in the example of the nucleus ^{198}Au [10] (see also ref. [11]).

There have also been developments over the last years in the theory of dynamical symmetries. For example, generalisations towards two different kinds of *partial* dynamical symmetry have been formulated [12,13]. And, very recently, a new type of dynamical symmetry was proposed that specifically deals with many-particle systems at or close to a phase transition [14,15] (see also ref. [16]).

Most of these applications of symmetry techniques concern nuclei close to the line of stability. But what about exotic nuclei?

2 Symmetry in exotic nuclei

Given the current direction of nuclear-physics research towards the investigation of rare isotopes, the role of symmetry in exotic nuclei should be examined. So far, symmetry-based applications to nuclei on the neutron-rich side are

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few. (An isolated example that speculates on the occurrence of new modes of excitation in nuclei with a neutron skin can be found in ref. [17].) In this contribution we focus on nuclei at the other side of the line of stability and specifically on self-conjugate ($N = Z$) nuclei. These are of interest for several reasons, but in particular because they are the only nuclear systems that might display effects of $T = 0$ pairing. Unlike the usual $T = 1$ pairing, which involves either neutrons or protons with antiparallel spins ($S = 0$), $T = 0$ pairing requires a neutron and a proton coupled to intrinsic spin $S = 1$. Whether nuclei at the $N = Z$ line display $T = 0$ pairing or not is still a hotly debated question and considerable effort is spent in designing, carrying out and analysing data from experiments that purport to study this question.

The ideal starting point to study these questions is, in fact, provided by an old algebraic model due to Flowers and Szpikowski, and known as the $SO(8)$ model [18]. The standard pairing model (which concerns $T = 1$ pairing between identical nucleons) was shown to have an $SU(2)$ dynamical symmetry [19]; $SO(8)$ is its generalisation that includes $T = 0$ as well as $T = 1$ pairing for a system of neutrons and protons. A group-theoretical analysis of the model shows that three analytical solutions of this generalised pairing Hamiltonian are possible: pure isoscalar pairing, pure isovector pairing and equal pairing strengths in the isoscalar and isovector channels. Although many results of interest can be extracted from this model, it remains nevertheless too schematic to be of relevance in nuclei. It can be made more realistic by including effects of the non-degeneracy of single-particle orbits generated by the nuclear mean field. Such an analysis has been performed recently with particular emphasis on the effect of the spin-orbit term [20]. It takes the $SO(8)$ model with non-degenerate single-particle energies as a starting point for a mapping onto a corresponding boson model in terms of $l = 0$ bosons with $s = 0$, $t = 1$ or $s = 1$, $t = 0$. The main conclusion of ref. [20] is that the spin-orbit term in the nuclear mean field destroys isoscalar superfluid correlations in self-conjugate nuclei.

The idea to map the shell model onto a corresponding boson model can be made more concrete as will be discussed in the remainder of this contribution. With $SU(4)$ supermultiplet symmetry and its generalisation to pseudo-$SU(4)$ (discussed briefly in sect. 3) as a starting point, a boson model is constructed (sect. 4) that includes $l = 0$ and $l = 2$ bosons with $s = 0$, $t = 1$ or $s = 1$, $t = 0$ (IBM-4) and hence provides a natural framework to study isoscalar and isovector pairing. First results of IBM-4 in the 28–50 shell are presented. The boson Hamiltonian is derived microscopically from a realistic shell-model Hamiltonian through a mapping that relies on the existence of approximate shell-model symmetries. Applications are presented for the odd-odd $N = Z$ nuclei from ^{58}Cu to ^{70}Br.

3 Pseudo-SU(4) symmetry in pf-shell nuclei

Except in p-shell nuclei, LS-coupling is not an appropriate classification scheme since L and S are badly broken by the strong spin-orbit term in the nuclear mean field. Hence Wigner's elegant supermultiplet model [2], based on $SU(4)$ symmetry, can only be applied to the very light nuclei. Nuclei beyond $^{56}_{28}$Ni$_{28}$, however, can be classified in terms of a pseudo-$SU(4)$ symmetry [21,22] leading to the following (approximate) labelling of shell-model states:

$$|n(\widetilde{\lambda\mu\nu})\tilde{\alpha}\tilde{L}\tilde{S}JM_J;TM_T\rangle, \tag{1}$$

where n is the number of nucleons in the valence shell which is taken to be the entire pseudo-oscillator shell (*e.g.*, pseudo-sd). The total angular momentum and isospin of all nucleons are J and T, respectively, and these quantum numbers are conserved by any rotationally invariant, charge-symmetric Hamiltonian. In addition, the total pseudo-orbital angular momentum $\tilde{L}$ and the total pseudo-spin $\tilde{S}$ are conserved, which result from the separate coupling of all individual pseudo-orbital angular momenta $\tilde{l}_i$ and pseudo-spins $\tilde{s}_i$ [23,24]. The $(\widetilde{\lambda\mu\nu})$ are the labels associated with pseudo-$SU(4)$ in direct analogy with Wigner's supermultiplet labels. Finally, $\tilde{\alpha}$ denotes any remaining label necessary for a full characterisation of the states in pseudo-orbital space.

In ref. [22] it was shown that the pseudo-$SU(4)$ classification (1) is relevant for nuclei at the beginning of the 28–50 shell; towards the middle of this shell the $g_{9/2}$ orbit starts playing an important role and the assumption of the dominance of the $pf_{5/2}$ orbits breaks down. The procedure for testing the validity of pseudo-$SU(4)$ is similar to the one followed by Vogel and Ormand [25], except that it is done for pseudo- instead of standard-$SU(4)$, and is explained in ref. [22].

4 The isospin-invariant boson model IBM-4

The IBM-4 [26] is the most elaborate version of the interacting boson model (IBM) of Arima and Iachello [7]. The bosons of IBM-4 are assigned an orbital angular momentum l which can be either $l = 0$ or $l = 2$. In addition, they are labelled by an intrinsic spin s and an isospin t, for which the combinations $(s,t) = (0,1)$ and $(1,0)$ are retained. The bosons represent correlated fermion pairs and the choice of bosons is dictated by the requirement that they should describe nuclear excitations at low energy. The particular choice made in the IBM-4 can be justified by considering the spectrum of two nucleons in a harmonic oscillator shell interacting via a delta-force. In the absence of a spin-orbit term in the mean field, this interaction results in low-energy states labelled by L, S, and T quantum numbers that have the same values as those of the bosons of IBM-4. Another justification of the choice of bosons in IBM-4, already emphasised by Elliott and Evans [26], is that it allows a classification in which there appears an $SU(4)$ algebra that is to be associated with Wigner's supermultiplet algebra (or its "pseudo" equivalent).

As regards applications, the set of bosons of IBM-4 is particularly well adapted to deal with even-even and odd-odd $N \approx Z$ nuclei. This was demonstrated for nuclei in the sd-shell [27,28] but a full test of the IBM-4

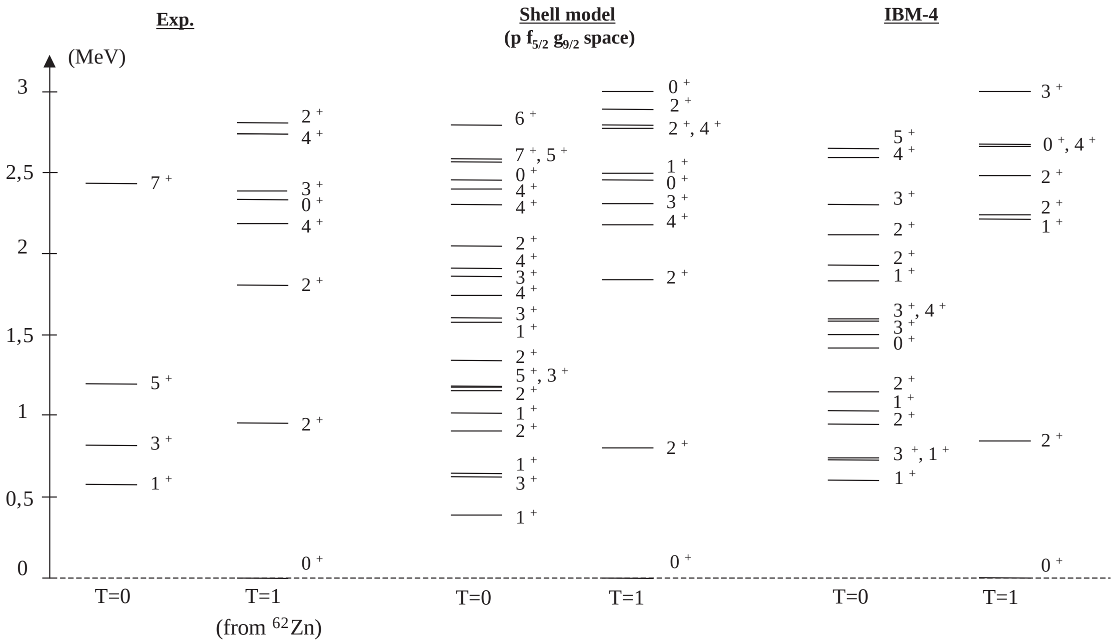

Fig. 1. Experimental, shell model and IBM-4 spectra of $A = 62$ nuclei. The experimental energies of levels with $T = 0$ are taken from ^{62}Ga, while those with $T = 1$ are deduced from the isobaric analogue states in ^{62}Zn.

remains desirable since Halse *et al.* projected onto a symmetric IBM-1 subspace and no calculations have been done yet in the complete IBM-4 space. The mass region of primary interest here is the first half of the 28–50 shell. Shell-model calculations in the full $pf_{5/2}g_{9/2}$ space are feasible for nuclei just beyond $^{56}_{28}$Ni$_{28}$ but they become increasingly difficult for the heavier isotopes. One thus enters a mass region where the full shell-model calculations are, if not impossible, at least arduous and where the IBM-4 offers a viable alternative.

The boson energies and the boson-boson interactions are determined with the usual procedure due to Otsuka, Arima and Iachello, referred to as OAI [29], which consists of the calculation of matrix elements of a realistic shell-model Hamiltonian between one- and two-fermion states and their mapping onto corresponding boson matrix elements. With reference to [30] for full details, here only the following important role played by symmetries is emphasised: they provide quantum numbers through which the correspondence between both spaces can be established. In addition, symmetries in the shell model greatly reduce the non-orthogonality of fermion-pair states and therefore simplify the construction of the boson Hamiltonian [31]. This is the reason why the starting point of the mapping is a shell-model calculation that preserves various symmetries.

The first test of the IBM-4 Hamiltonian thus derived is the three-boson nucleus $^{62}_{31}$Ga$_{31}$. Figure 1 shows the known experimental levels [32] together with the shell-model [32] and the IBM-4 results. (We should mention that there are some conflicting spin assignments regarding this nucleus [33].) Both shell model and IBM-4 predict a 0^+ $(T = 1)$ ground state and a 1^+ $(T = 0)$ first-excited state. Note that this represents an inversion with respect to the order in $^{58}_{29}$Cu$_{29}$ which agrees with the data. Given that no free parameter is introduced in the IBM-4 calculation, the agreement for the isoscalar levels between shell model and IBM-4 can be called remarkable and a near one-to-one correspondence between levels can be established, the exceptions being higher-spin (5^+ and 7^+) shell-model states which are absent from the IBM-4 because it does not include high-spin $T = 0$ bosons. Note also a low-lying 0^+ state in the IBM-4 calculation which, since the shell-model counterpart is much higher in energy, might have an important spurious component. Experimentally, excited states in ^{62}Ga were located for the first time [32] in an experiment which populated the nucleus through a fusion-evaporation reaction. However, this type of study of $N = Z$ nuclei in this region is difficult, requiring high experimental sensitivity, and yields information only on the yrast structure. The vast majority of $T = 0$ states predicted by the shell model or the IBM-4 thus remains to be verified.

A similar situation applies to $^{66}_{33}$As$_{33}$ although in this case the population was via isomeric states [34]. Only a few states have been identified and without unique spin assignments. The population of excited states in $^{70}_{35}$Br$_{35}$ was reported but subsequently withdrawn [35]. Recent experiments should soon clarify the situation in this nucleus [36]. At this moment, however, a meaningful comparison with the IBM-4 results of fig. 2 is still premature.

Turning now to the $T = 1$ states, in ^{62}Ga, one notes more levels in experiment and the shell model as compared to the IBM-4. This deviation grows in the heavier nuclei ^{66}As and ^{70}Br, where the $T = 1$ energies can be taken from the experimental level schemes of the isobaric

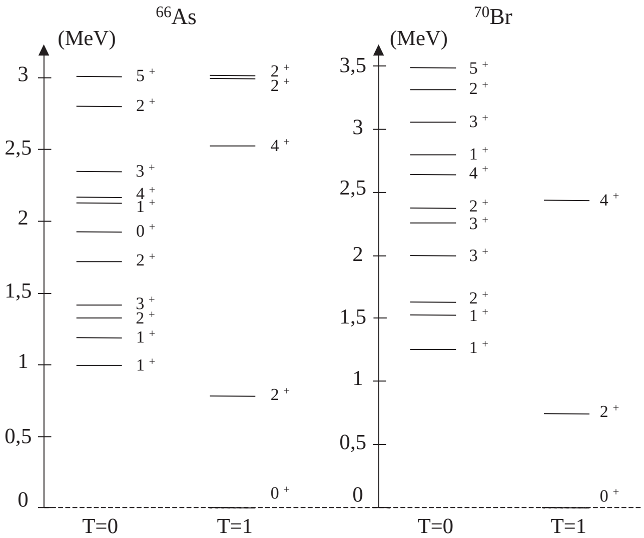

Fig. 2. Spectra of ^{66}As and ^{70}Br predicted by IBM-4.

analogues in $^{66}_{32}\mathrm{Ge}_{34}$ and $^{70}_{34}\mathrm{Se}_{36}$. In particular, one notes the absence from IBM-4 of a second 2^+ level at the observed experimental energy ($E_x \approx 2$ MeV). In a corresponding IBM-3 analysis (which is feasible for the $T = 1$ subspace of $N = Z$ nuclei) this state is correctly reproduced but only after allowing a microscopically dictated boson-number dependence of the Hamiltonian. This deficiency of the current calculation for isovector states can thus presumably be traced back to the constancy of the boson Hamiltonian for all nuclei shown and indicates the need to derive a boson-number dependence in IBM-4 too.

5 Conclusion

The present results illustrate the predictive power of the IBM-4. In particular, the $0^+(T = 1)$–$1^+(T = 0)$ splitting is correctly reproduced in the known cases, ^{58}Cu and ^{62}Ga, and is predicted to be about 1 MeV in ^{66}As and ^{70}Br where it is not well established experimentally and where a shell-model calculation currently is still difficult. Given the complexity of the IBM-4 Hamiltonian derived from the shell model, it will now be of interest to extract its essential features that lead to this behaviour.

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